Information Discreteness and the Market-State Effect on Momentum

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Abstract

Momentum profitability depends on the market state, with protracted market gains heralding momentum gains and prolonged market declines ushering momentum losses. This study documents that information discreteness fully subsumes the market-state effect on momentum profitability. Regardless of the market state, high information discreteness predicts both short- and long-term losses, whereas low discreteness predicts momentum profits, consistent with the frog in the pan (FIP) hypothesis. Moreover, momentum crashes are ushered by periods marked by extremely high discreteness levels. This study employs two novel discreteness measures: a time-varying adaptation of the measure previously employed to document the cross-sectional effect of discreteness on momentum and a crude but easily accessible outlier-based gauge of the incidence of highly discrete information shocks.

Keywords: Anomalies; momentum; information discreteness; market-state; equities; investor attention; predictability.

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Introduction

Starting from the seminal study (Cooper, Gutierrez and Hameed, 2004, henceforth CGH), it is well known that short-term momentum gains stem exclusively from portfolios formed in months following prolonged aggregate market gains. In contrast, protracted negative market returns herald short- and long-term momentum losses. These phenomena are called the market-state effect on momentum, taken collectively.[1]

As documented in CGH, the effect of the market states on momentum profitability cannot be explained by the usual risk adjustments or by controlling for macroeconomic conditions. This study proposes an explanation for these longstanding puzzles by showing that the market-state effect is driven by information discreteness and, thus, ultimately, by investors' limited attention.

The link between attention and momentum is offered by the Frog in the Pan (FIP) hypothesis (Da, Gurun, Warachka, henceforth DGW, 2014).[2] The conjecture is that investors pay more attention to news reaching the market through large discrete signals than to information diffusing continuously (for an equal level of information content). Using a novel measure of information discreteness (henceforth, the ID_{DGW} measure), DGW concludes that, due to investors' limited attention, news diffusing continuously originates the prolonged price trends on which the momentum strategy capitalizes, whereas information diffusing via discrete information shocks yields low momentum gains as news are promptly incorporated into prices.[3]

I rely on two novel aggregate measures of informational discreteness, later described, to document a strong predictive power of discreteness for momentum profitability. [4]

[1] Li and Galvani (2018) found an equivalent market-state effect on momentum for US corporate bonds.

[2] The FIP conjecture belongs to a stream of literature exploring the implications of investors' cognitive limitations and specifically investors' attention (see (e.g., Gabaix, 2019) for a review).

[3] Recently, Huang et al. (2022) rely on the ID_{DGW} measure to analyze the lead-lag relationship between firms in a customer-supplier relationship and find corroborating evidence to the view that information received via large (discrete) signals might act as an attention trigger and prompt rapid price adjustments.

[4] For ease of exposition, here and in the following, momentum refers to strategies with the holding period horizons of three-, six-, and twelve-month, taken collectively.

Specifically, the momentum strategy is unprofitable following periods marked by severe discreteness, whereas momentum gains exclusively follow periods with more continuously diffusing news. These effects are particularly compelling when severe discreteness involves news generating negative price trends (i.e., discreteness in bad news). Further, there is a broadly monotonic negative relationship between discreteness and momentum profitability along the time dimension, a finding echoing the conclusion in DGW for the ID_{DGW} cross-sectional measure. These results link the time variations of momentum profitability to investors' attention (through information discreteness).

According to the FIP hypothesis, the interpretation of these findings is clear. Investors pay particularly keen attention to highly discrete news, and the prompt incorporation of information into prices halts or reverses the price trends on which the momentum strategy capitalizes. The implication is that market phases in which news arrives through highly discrete signals are associated with momentum losses, whereas periods featuring particularly low discreteness levels yield the most robust momentum gains.

Both discreteness and lagged market returns predict momentum profitability. However, the predictive power of the market can be reduced to that of information discreteness once we note the market states are heterogeneous in terms of average discreteness. The results show that, on average, information causing strong price trends reaches investors via discrete signals when market returns are low and through continuous signals in normal market conditions. Moreover, bad news tends to diffuse continuously in buoyant markets, whereas good news arrives with high discreteness. Given these results, market states might predict momentum returns because of heterogeneity in their average level of information discreteness. If this is the case, the market-state effect on momentum should vanish once we control for discreteness levels.

Consistently, a double sort of momentum returns on discreteness and market states provide strong evidence that the former subsumes the latter. Specifically, when highly discrete information shocks are prevalent, there are no momentum gains for all the aggregate conditions summarized by the market states. In contrast, there are substantial momentum gains, regardless of the market state, when the discreteness of news is more moderate. The effect is particularly clear for discreteness in bad news, suggesting the market states proxy the prevalence of highly discrete adverse information shocks.

Another puzzling aspect of the market-state effect is that momentum profitability peaks in the central state (i.e., the median market quintile), as already documented in CGH. The results show that average discreteness is particularly low in the central state, a finding that is consistent with the FIP hypothesis, according to which particularly low discreteness entails strong momentum gains. Further, momentum profitability vanishes even in the median state when discreteness is high, a finding that is consistent with discreteness rather than the market state, driving the time variations of momentum returns.

Building on the insights provided by the FIP hypothesis, this study provides an explanation for the occasional instances in which momentum investing yields abysmal returns (Daniel and Moskowitz, 2016; Barroso and Santa-Clara, 2015). The results show that all the months in which momentum returns fall in the bottom one percentile (i.e., momentum crashes) are preceded by the arrival of highly discrete information. For instance, the most recent momentum crash of 2009 stems from bad news reaching the market with discreteness around the 98th discreteness percentile, a discreteness level not unexpected for those familiar with the events defining the Great Financial Crisis.

The assessment of the role of information discreteness in explaining the market-state effect requires an aggregate time-varying measure of discreteness independent from specific momentum portfolios. Since the ID_{DGW} measure is defined at the stock level, it is tempting to aggregate it over the cross-section to obtain a market-wide discreteness measure and then use it to compare discreteness over the market states. The difficulty is that the aggregation would combine ID_{DGW}-discreteness values that are not comparable, as they are potentially associated with different levels of both information content and discreteness.[5]

Mindful of this constraint, I develop a set of novel market-wide time-varying discreteness proxies, collectively called the Conditional Information Discreteness (CID) measures, which are time-varying extensions of the cross-sectional ID_{DGW}-discreteness proxy. The CID measures capture the incidence of high discreteness among stocks showing strong price trends.[6]

The ID_{DGW} and CID measures provide valuable insights into the origin of the momentum effect from the cross-sectional and dynamic perspectives, respectively. However, these measures are calculated on daily returns, which implies that there are markets for which they might not be readily available. For instance, infrequent trading in the corporate bond markets makes the evaluation of the ID_{DGW} measure problematic.

In view of this limitation, this study proposes a novel (and easily accessible) discreteness measure based on the incidence of monthly return outliers over the momentum strategy's formation period.[7] Outliers have already been used in the literature to gauge substantive information shocks (e.g., Odean, 1998; Frank and Sanati, 2018). This study proposes that outliers also provide insights into information discreteness and thus should predict momentum profitability, consistent with the FIP hypothesis.

The outlier-based measure shows a substantial predictive power for momentum gains, being able to differentiate periods heralding momentum gains and losses. Further, this crudely defined discreteness measure broadly subsumes the market-state effect.

In addition to proposing two novel information discreteness measures, this study contributes to four separate lines of research within the literature on momentum. First, the findings reframe the discourse about the time-dynamics of momentum profitability (e.g.,

[5] In fact, the ID_{DGW} measure is defined conditionally on individual stocks' cumulative returns over a given period (i.e., the momentum strategy's formation period), where the formation period return gauges the information content affecting the stock price.

[6] To briefly outline the procedure, I pool the formation-period returns and rank them into twenty quantiles to obtain (intertemporal) groups with homogeneous information content. Within each return band, I identify the returns with high ID_{DGW} discreteness. Discreteness in a given month is captured by the incidence of high discreteness stocks in the cross-section.

[7] Specifically, the percentage of stocks with at least one monthly return outlier over the formation period.

Barroso and Santa-Clara, 2015; Daniel and Moskowitz, 2016; Cooper et al., 2004; Li and Galvani, 2018) by shifting the focus from market conditions to information discreteness and thus investors' attention. Further, this study provides an information-based explanation of the large dramatic declines in momentum profitability. Specifically, momentum crashes are ushered by periods marked by extreme information discreteness and thus heightened investors' attention.

By analyzing the differences in momentum for bonds traded by institutional and retail investors, Li and Galvani (2021) conclude that differential information diffusion rates yield distinct momentum return patterns, with slow information originating strong momentum gains. Albeit these conclusions are drawn for corporate bonds, the effect of information diffusion speed was already theorized in Hong and Stein (1999a) and outlined for equities in Hong et al. (2000). DGW adds the insight that equity momentum originates from those information shocks that draw scant investors' attention, with the momentum strategy capitalizing on information diffusing through continuous signals. This study shows that high levels of information discreteness usher disappointing momentum profits. Hence, the link between slow information diffusion and momentum gains documented in Li and Galvani (2021) might be viewed through the lens of information discreteness, with continuously diffusing news spreading slowly due to scant investors' attention. Conversely, fast information diffusion speed might be explained by news reaching the market via discrete signals, which trigger investors' attention and are quickly imbued into prices.

By documenting substantial similarities, in terms of momentum predictability, between the CID measures and the incidence of extreme returns, this study adds supporting evidence to the conjectured link between the magnitude of equity returns and investors' behaviours (e.g., Odean, 1998; Frank and Sanati, 2018; Li, 2023). The same equivalence also suggests a tentative explanation for the curious finding that negative return outliers falling in the momentum strategy's formation period disproportionally weaken momentum profitability for corporate bonds (Galvani and Li, 2023). Namely, if extreme negative returns attract investors' attention, they should weaken momentum returns, according to the FIP hypothesis.

The CID measures' ability to predict instances in which momentum returns drop spectacularly (Daniel and Moskowitz, 2016; Barroso and Santa-Clara, 2015) is a further validation of the FIP hypothesis. Therefore, the insights offered by this study might be helpful in designing a momentum strategy that adjusts for high-discreteness information, thus contributing to the growing line of research aiming to improve the performance of momentum investing (e.g., Moreira and Muir, 2017; Cederburg et al., 2020).

1 Data and Methodology

The data consists of all NYSE and AMEX ordinary stocks listed on the CRSP monthly and daily files. This study relies on a sample period (1925-2022) that is about three decades longer than that used in CGH. These additional decades include two major stock market crashes, namely the 2008-2009 financial crisis and the Covid-19 pandemic. These extreme scenarios offer the opportunity to re-evaluate the findings of CGH in a sample that includes two market episodes showing equity devaluations comparable to those observed in the 1929 stock market crash. As done in DGW, in each month, negative book-to-market stocks and stocks with prices under $5 are excluded from the sample.[8]

1.1 Momentum Strategies

This study's empirical approach to the evaluation of the market-state effect on momentum closely follows CGH to foster comparability.[9] The analysis focuses on the familiar

[8]The conclusions of this study remain virtually unaltered when excluding from the momentum strategies stocks with prices under $1 in the last month of the formation period, as done in CGH. The results are in the Appendix in Section A.1.

[9]The main methodological departure from CGH is the use of value-weighted momentum portfolios. Using equally weighted portfolios does not alter this study's conclusions.

six-month formation-period momentum strategy (Jegadeesh and Titman, 1993), with a skip month.[10] In each month t, stocks are ranked into deciles based on the cumulative returns between $t-5$ to $t-1$. A value-weighted portfolio of the stocks in the top and bottom deciles are the long and short sides of the momentum strategy, respectively. For a stock to be included in the momentum portfolio, the returns for all the months of the strategy's formation period must be available. Holding period returns are calculated starting from $t+1$.

To minimize the effect of idiosyncratic risk, the cumulative returns of momentum strategies for which the winner or loser portfolios include fewer than five stocks at formation are marked as missing. Removing this filter, which mainly affects the early part of the sample, does not alter this study's conclusions. However, this restriction explains the variations in the number of observations falling in momentum cumulative return quantiles in some tables.

Holding period cumulative returns are calculated over horizons ranging from one to five years. The average cumulative returns between months 13 and 60 are reported separately to discuss long-term momentum reversals, as done in CGH. The risk-adjusted cumulative returns of the momentum strategies are calculated as in CGH, for the single-factor market model and for the three-factor model of (Fama and French, 1993).[11] I briefly review the procedure to obtain risk-adjusted momentum profits and refer the reader to CGH for details. For each month in the holding period, the strategy's raw return is regressed on the appropriate risk factors and a constant. The corresponding adjusted return is the difference between the raw momentum return and the predicted return obtained by multiplying the estimated loading and the contemporaneous factor returns (excluding the constant). For a given formation month and holding period, the holding-period risk-adjusted monthly return is the average of the obtained risk-adjusted monthly returns

[10]Following CGH, the results of this study are presented for the 6m. formation-period strategy. Relying on 12-month formation periods yields very consistent conclusions, as shown in the online appendix.

[11]The CAPM is estimated using the CRSP value-weighted market index (with dividends) for consistency with the identification of the market states.

over the holding period.

1.2 Market States

This study's results are presented for market states identified by quintiles of the distribution of the three-year market average returns.[12] CGH defines the UP and DOWN market states by partitioning the same distribution by the zero-return threshold. Specifically, they define a month as in the UP or DOWN state depending on the sign of the average monthly market return over the preceding three years. The zero-return cut-off corresponds to about the 13.5th percentile of the three-year average monthly market return distribution. The implication is that the UP market state lumps together almost 90% of the months in the sample period. The results of this study are introduced for the quintile-based market states, to gain insights into the link between information discreteness at a more granular level. Section A.1 in the appendix discusses the interaction between discreteness and market states defined by the zero cutoff. The two approaches yield consistent conclusions.

The bottom quintile of the three-year market average monthly returns is termed the DOWN market state for ease of exposition.[13] In contrast, the CGH market states are termed 0-UP and 0-DOWN, to emphasize the use of the zero threshold.

1.3 Stratified Averages

Several of the results of this study are presented in terms of stratified averages of momentum monthly holding-period returns. These averages are obtained as in CGH, that is, by regressing returns over the stratifying categories with no constant. For instance, the average momentum monthly holding-period returns are stratified on the market states by regressing the returns over the five 0/1-categorical variables identified by the mar-

[12]Employing the one-year average does not alter this study's conclusions, as already verified by CGH for the market-state effect on momentum. Following CGH, the market index is the CRPS value-weighted market index, with dividend reinvestment.

[13]The threshold identifying the bottom quintile is 36 bps.

ket states, with no constant. The market categories are assigned to each holding period monthly return by the momentum portfolio's formation month.

Following CGH, since the holding-period returns are overlapping, standard errors are corrected for heteroskedasticity and autocorrelation (HAC) with lags equal to the number of overlapping months in the holding-period window. The same methodology is used to stratify momentum holding-period monthly returns over the various information discreteness variables.

1.4 The DGW ID measure

The discreteness measure proposed in DGW for stock i in month t is defined as:

$$ID_{DGW} = sign(PRET) * (\%neg - \%pos)$$

where $sign$ is the sign function, PRET is the cumulative return of the stock over the momentum strategy's formation period, and the percentages % neg and % pos are the raw-terms percentages of negative and positive daily returns over the formation period for the same stock.The ID_{DGW} measure ranges between $+1$ and -1, where $+1$ indicates the highest level of information discreteness.[14]

For emphasis, the ID_{DGW} measure is defined under the assumption that the cumulative return of the stock (i.e., PRET) is a proxy for the information content affecting the stock price over the formation period. Comparisons (e.g., ranking) in terms of the ID_{DGW} measure can be performed across stocks only if their formation-period return is similar, as doing otherwise would conflate the effects of the discreteness and content of the information shock. Readers can refer to DGW for several validations of the conclusion that the

[14] To illustrate, consider a stock included in the loser portfolio for which $sign(PRET)$ is negative. If the cumulative return is obtained by only one daily negative return, then the measure takes the value $+1$. If the same cumulative return is obtained by a series of negative daily returns and no positive daily return, the value of ID tends to -1.

ID_{DGW} measure gauges investors' attention.[15]

1.5 The Conditional ID Measures

I extract from the ID_{DGW} measure three gauges of aggregate information discreteness, collectively called the Conditional Information Discreteness (CID) measures. For each month t, I calculate the formation-period return of each stock in the cross-section, where the formation period is from $t-5$ to $t-1$.[16] Next, I partition the resulting intertemporal sample of formation-period returns into twenty quantiles, where each quantile groups formation-period returns with comparable information shocks.[17] This step results in twenty intertemporal formation-period return subsamples with homogeneous information content. I call the elements of this partition return bands. Note that the formation-period returns included in each band do not need to overlap along the time dimension.

Within each return band, formation-period returns are ranked on ID_{DGW}-discreteness and then identified as high discreteness if they fall in the band-specific discreteness tercile.[18] At each t and within each return band, I calculate the percentage of the formation-period returns (i.e., returns from $t-5$ to $t-1$) with high discreteness, where the percentage is over the number of formation-period returns falling in the return band in the cross-section of month t. For the return band b and formation periods from $t-5$ to $t-1$, this variable is denoted by $HighID_{b,t}$.

The percentage $HighID_{b,t}$ gauges the average likelihood with which the information originating the returns from $t-5$ to $t-1$ in return band b reached the market with high

[15]For instance, the authors compare the predictive power of ID_{DGW} for momentum profitability for firms with different levels of attention constraint as proxied by firm-level institutional ownership, market capitalization, analyst coverage, and the degree of media coverage. DGW also shows that conditional on formation-period returns, the predictive power of the ID_{DGW} measure for momentum is not subsumed by firm characteristics that the literature has linked to the strength of the momentum effect (e.g., the disposition effect and idiosyncratic volatility).

[16]For clarity, month t is the formation month (the skip-month) of the momentum strategy.

[17]The high level of fineness of the partition is driven by the wide range of six-month returns in the sample. Finer partitions yield equivalent results.

[18]Using quintiles or the median to identify high-ID_{DGW} formation periods yields consistent results (untabulated results).

levels of discreteness. For example, focusing on the return band of the lowest cumulative returns (i.e., $b = 1$), the percentages $HighID_{1,t}$ informs us about the likelihood with which bad news falling between $t-5$ to $t-1$ diffuses with high discreteness.

Given a band b, the time-variations of $HighID_{b,t}$ bear an intuitive interpretation. For instance, among the formation periods in the highest return band $b = 20$, the variable $HighID_{20,t}$ gauges the likelihood (over time) with which the strongest price appreciations (i.e., the most positive good news) are associated with high discreteness. For instance, a value of 1.5 for $HighID_{20,t}$ at time t means that 1.5% of the stocks in the cross-section t experienced very strong positive price trends caused by highly discrete news during the formation period.

I define the aggregate time-t conditional information discreteness measure as the average of the percentages of high discreteness formation-period returns falling in the top and bottom three return bands. [19] I term this measure Conditional Information Discreteness (CID) and denote it by $CID_{pos,neg}$. I also define the corresponding measures for negative and positive strong price trends separately, which are denoted by CID_{neg} and CID_{pos}.[20] These measures capture information discreteness in bad and good news, respectively.[21] The correlations of the CID measures are reported in Table 3.

For emphasis, the stocks included in the momentum portfolio at time t are identified by the top and bottom deciles of the formation-period returns from $t-5$ to $t-1$. In contrast, the identification of high-discreteness formation-period returns is based on return bands, where the bands are not linked to a specific cross-section.

Untabulated results show that, on average, over time, about 70% of the formation period returns of the stocks in the loser and winner portfolios fall into the three bottom and

[19]Hence, for each t, the variable $CID_{pos,neg}$ is the average of the variables $HighID_{b,t}$ over the return bands $b = 1, 2, 3, 18, 19, 20$. The subscript t is omitted from the CID measure for notation simplicity.

[20]CID_{neg} is the average of $HighID_{b,t}$ over $b = 1, 2, 3$. The measure CID_{pos} is defined analogously for $b = 18, 19, 20$.

[21]The main conclusions of this study remain qualitatively unaltered when we define the CID measures based on the top and bottom two return bands, or the 20th and 1st bands.

top return bands, respectively.[22] I visualize these insights in the panels of Figure 1. The left-side panel plots the average (over time) percentage of losers and winners by return bands. The plot illustrates how the mid-range return bands are unlikely to include a stock selected into the momentum portfolio, consistent with the theoretical prediction Hong and Stein (1999a) that momentum capitalizes on the continuation of strong price trends. The right-side panel plots the percentage of losers and winners falling in the bottom- and top-three return bands, respectively, sorted by market states. The insight is that the percentage of winners and losers with formation-period returns marked by strong price trends (i.e., falling in bands $b = 1, 2, 3$ and $b = 18, 19, 20$, respectively) remains large across market states, with little variation.

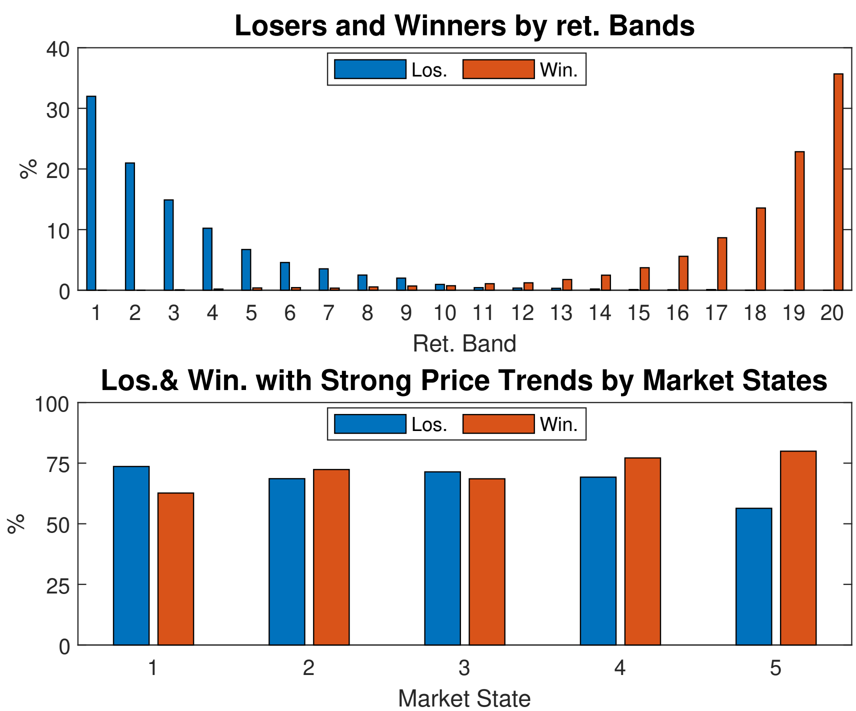

Figure 1: The sample of six-month cumulative returns is partitioned into twenty quantiles, thus creating twenty return bands. The top panel of the figure plots the average (over time) percentage of winners and losers with formation-period returns falling in each return band. The bottom panel shows the average (over time) percentage of losers and winners falling in the bottom- and top-three return bands, respectively, sorted by market states (quintiles).

[22] About half of these fall in the 1st and 20th return bands, respectively.

An alternative to relying on discreteness in strong price trends is to consider the cross-sectional average of the incidence of high discreteness over all the return bands (i.e., the average of $HighID_{b,t}$ over $b = 1, ..., 20$). Most of the conclusions of this study are validated by the use of this alternative measure. However, this measure shows a weaker predictive power for momentum returns than the CID measures. The reason is that when averaging over a larger number of return bands, the share of winners and losers defining the aggregate discreteness measures decreases, which weakens the insights offered by the conditional discreteness measure on the origin of the momentum effect.

At the outset, the commonalities between the CID measures and the momentum profitability reside solely on the observation that the assets included in the momentum portfolio are likely to show strong price trends during the momentum strategy's formation period. Conversely, stocks with strong price trends contribute to the CID measures, but only if they also show high discreteness, regardless of their inclusion in the winner and loser portfolios.

1.6 Discreteness and Return Outliers

This study explores a crude measure of information discreteness based on return outliers, based on the intuition that extreme price movements are a response to the arrival of highly discrete news on the intuitive notion that return outliers are the response to severe information shocks (e.g., Li, 2023; Odean, 1998; Frank and Sanati, 2018) that reach the market with high discreteness. Specifically, I explore the percentage of stocks with at least one outlier over the formation period as a measure of discreteness. Extreme returns are identified by the 1st and 99th percentiles of the monthly return sample, at about -38.9% and 55.5%, respectively.[23]

[23] For each formation month t, the measure is the percentage (over the cross-section) of stocks that show at least one extreme return in the formation period. This percentage is evaluated for the pool of stocks from which winners and losers are selected.

1.7 Summary Statistics

Table 1 reports basic summary statistics for the discreteness variables and the three-year market returns. On average, a relatively small fraction, at about 6%, of stocks in the cross-section is associated with formation periods linked to high degrees of information discreteness. Table 3 reports correlations for the discreteness variables proposed in this study and the market three-year average monthly returns.

Table 1: Summary Statistics

	Mean.	25th prc.le	50th prc.le	75th prc.le	max	min	stdev
ID_{DGW}	-0.060	-0.108	-0.049	0.000	1	-1	0.099
$CID_{pos,neg}$	1.686	0.901	1.371	2.070	9.563	0.025	1.305
CID_{pos}	1.587	0.206	0.875	1.965	14.525	0	2.265
CID_{neg}	1.784	0.540	1.147	2.219	18.306	0	2.112
Out	6.623	1.230	3.390	6.382	70.428	0	10.367
MKt	0.912	0.582	1.001	1.395	3.525	-4.060	0.869

The table reports basic summary statistics for the stock-level discreteness variables ID_{DGW} calculated as in Da et al. (2014), and the CID monthly discreteness measures, $CID_{pos,neg}$, CID_{pos}, and CID_{neg}, which capture the incidence of high discreteness for good and bad news ($CID_{pos,neg}$), for good news ($CID_{pos,neg}$), and for bad news (CID_{neg}). The table also report the summary statistics for the outlier-based discreteness variable and for the market three-year average monthly returns (in percentage terms).

Table 2: Correlations

	MKT	$CID_{pos,neg}$	CID_{pos}	CID_{neg}	Out
MKT	1.00				
$CID_{pos,neg}$	-0.28	1.00			
CID_{pos}	-0.05	0.56	1.00		
CID_{neg}	-0.28	0.63	-0.29	1.00	
Out	-0.41	0.71	0.44	0.41	1.00

Table 3: The table shows the correlation matrix for the CID measures, the outlier-based discreteness variable, and the three-year average monthly market returns. For each t, the CID and outliers measures are calculated from returns between $t-5$ to $t-1$, whereas the market average return is over $t-36$ and $t-1$.

2 The Market-State Effect on Momentum and Discreteness

2.1 The Market-State Effect on Momentum

To set the stage, I establish the relevance of the market-state effect on momentum in the sample. As shown in Table 4, there are significant momentum returns for the three-, six-, and twelve-month investment horizons only following the top four quintile-based market states. DOWN markets are followed by momentum raw losses and risk-adjusted insignificant returns. As already noted in CGH, momentum profits peak in the median market quintile.[24]

2.2 The Market-State Effect on the ID_{DGW} measure

As already documented in DGW, momentum profitability decreases in the incidence of high information discreteness, consistent with the FIP hypothesis. However, once we disaggregate the sample along the UP and DOWN market states, we find that this relationship holds only in UP markets, whereas even the portfolios with the lowest levels of information discreteness fail to yield momentum gains in DOWN markets. The results are in the Appendix, Section A.2, Tables Table 17, 18 and 19.[25]

Under the assumption that low information discreteness drives momentum gains (i.e., the FIP hypothesis), a possible explanation of the absence of momentum profits in DOWN markets for all ID_{DGW}-discreteness quintiles is that a DOWN market increases the level of information discreteness for all the cross-section so that even stocks in the lowest quintile in ID_{DGW} fail to yield momentum profits, a possibility for which this study offers corroborating evidence, as we shall see next.

[24] Table 13 in Section A.1 show the corresponding results for the 0-DOWN and 0-UP market states.

[25] In keeping with the approach of CGH to evaluate the market-state effect, this study relies on the 6-m. formation-period strategy with holding periods ranging from one month to five years. In contrast, da2014frog relies on 12-m. formation and holding periods. The conclusions of this study do not vary when considering the 12-m. formation-period strategy.

Table 4: Market State Effect (Quintiles)

N.obs.	MRK Qunt.le	3m.	6m.	12m.	13-60m
		Panel A: Raw			
213	DOWN	-0.872**	-0.355	-0.215	-0.161**
214	2	0.69***	0.694***	0.553***	-0.148
213	3	1.52***	1.08***	0.741***	-0.155*
214	4	0.351	0.585***	0.288**	-0.167*
214	5	1.1***	0.727**	0.180	-0.248*
		Panel B: CAPM alphas			
213	DOWN	-0.678*	-0.213	-0.141	-0.242***
214	2	0.803***	0.771***	0.592***	-0.202**
213	3	1.58***	1.15***	0.776***	-0.218***
214	4	0.574**	0.716***	0.358**	-0.231***
214	5	1.15***	0.767**	0.2	-0.294**
		Panel C: FF alphas			
213	DOWN	-0.522	-0.046	0.034	-0.133
214	2	0.797***	0.754***	0.602***	-0.131*
213	3	1.72***	1.25***	0.851***	-0.168**
214	4	0.671***	0.897***	0.534***	-0.208***
214	5	1.34***	0.944***	0.391	-0.237**

The table reports the stratified average over the market states (quintiles) for the post-formation period unadjusted and risk-adjusted monthly returns on the 6m. formation-period momentum strategy for the holding periods of 3, 6 and 12 months. The table includes the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

2.3 Variation of the Conditional ID Measure on Market States

According to the FIP hypothesis, high levels of discreteness are detrimental to momentum returns. Therefore, the variations of momentum profitability over the market states could stem from market states' heterogeneity in discreteness. To explore this possibility, I evaluate the excess incidence of high discreteness in the bottom and top three return bands individually. Excess discreteness is defined relative to a benchmark state, which is, in turn, the DOWN market, the median (central) market quintile, and the top market quintile. Presently, I regress the variables $HighID_{b,t}$ for $b \in \{1,2,3,18,19,20\}$ over a constant and four market-state 0/1-dichotomous variables for the market quintiles other than the benchmark and a constant. I then repeat the analysis for the excess discreteness for the $CID_{pos,neg}$ and CID_{neg} measures.[26]

For brevity, the results of the regression-based evaluation of excess discreteness for the central and top market quintiles are relegated to Section A.3 in the Appendix.[27] Instead, I visually illustrate how discreteness varies over the market state in the top panel of Figure 2, which plots the incidence of high-ID stocks in the 1st and 20th return bands. These bands include the strongest negative and positive formation-period cumulative returns in the sample, respectively. The figure also shows the incidence of high discreteness in the central (i.e., 10th) return band, which captures more subdued price trends. Similarly, the bottom panel of Figure 2 shows the stratified averages of the conditional information discreteness measures $CID_{pos,neg}$ and CID_{neg} over market states.

As visualized by the figure (and confirmed by the formal evaluation of excess discreteness), the percentage of high-ID stocks is generally significantly larger in DOWN markets than in the other market states. This effect is more marked for the bottom three return bands, indicating that strong negative price trends in DOWN markets are particularly likely to be generated by information spreading with high discreteness. Consistent with

[26]Standard errors are adjusted to account for overlapping returns.

[27]Results are reported in Tables 20, 21, and 22 where the benchmark state are the DOWN, central and top market state, respectively.

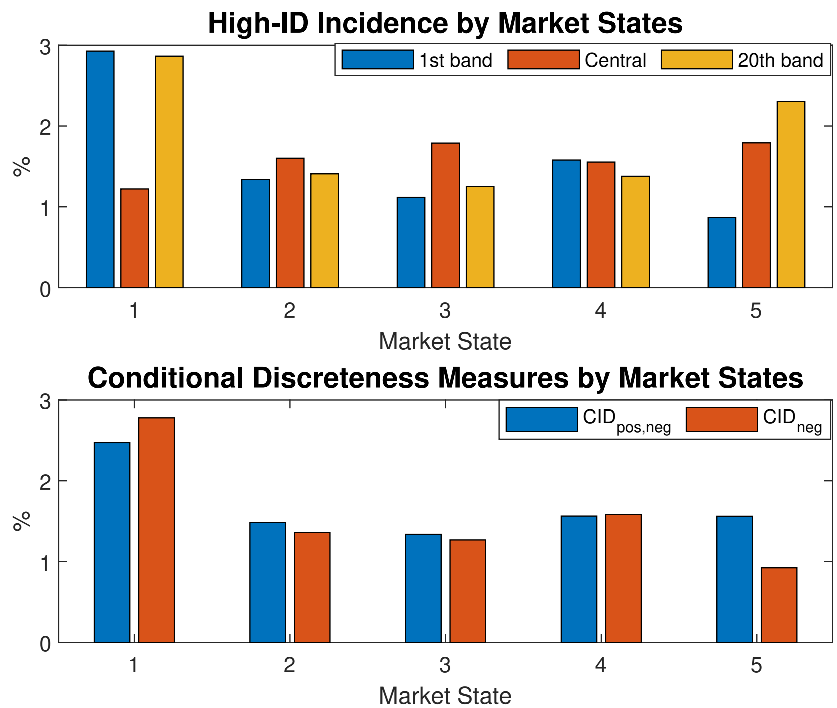

Figure 2: The sample of 6m. formation-period returns is partitioned into twenty return bands. In each of these bands, high ID stocks are identified using terciles of the ID_{DGW} distribution. The top panel shows the average percentage of stocks showing high discreteness in the 1st, 10th (central) and 20th return bands stratified by market states (quintiles). The bottom panel plots the averages on the market states of the conditional discreteness measures defined on the three top and bottom return bands ($CID_{pos,neg}$) and on the bottom three return bands (CID_{neg}).

this finding, the CID_{neg} measure reaches its highest levels in DOWN markets.

The median state generally shows lower levels of information discreteness relative to the other market states, an observation mirrored by lower levels in the $CID_{pos,neg}$ and CID_{neg} measures. An exception is a higher incidence of information discreteness in negative price trends in the central state than in the top market state. This is because the top market quintile is characterized by a very low incidence of high discreteness among negative price trends. Interestingly, the top market state shows high discreteness levels in positive price trends relative to the other market quintiles. This contrasting effect is captured by the CID_{neg} measure being lower in the top market state than in all the other four market quintiles and by a level of the $CID_{pos,neg}$ measure that is lower than in the DOWN market state but higher than in the median market state.

In summary, the results indicate that news causing strong price trends reaches the market via discrete signals in DOWN markets but via continuous signals in normal market conditions (i.e., the median state). During buoyant markets, news causing strong negative price trends tends to diffuse continuously, whereas good news arrives with high discreteness.

According to the FIP hypothesis, high (low) levels of discreteness are detrimental (beneficial) to momentum profitability. Therefore, the variation in information discreteness across the states provides a potential explanation of the market-state effect on momentum. Whether it is the market state or discreteness that explains momentum returns' dynamics is the empirical question explored in this study.

3 The Conditional Information Discreteness Effect on Momentum

The FIP hypothesis maintains that high (low) levels of information discreteness weaken (strengthen) momentum returns. From this perspective, the more the price trends of the stocks included in the momentum portfolio are likely to be generated by discrete information shocks, the weaker the momentum returns should be, on average. The CID measure is an aggregate time-varying gauge of the incidence of high levels of information discreteness for stocks with strong price trends. Hence, this discreteness measure should have some predictive power for momentum profitability.

Table 5: Stratified Momentum Averages on $CID_{pos,neg}$-discreteness

N.obs.	CID Qunt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw			
213	1	1.21***	0.955***	0.689***	-0.134
214	2	1.37***	1.17***	0.771***	-0.121
213	3	0.636***	0.558***	0.361***	-0.133
214	4	0.171	0.206	0.0745	-0.207**
213	5	-0.615	-0.158	-0.355	-0.288***
		Panel B: CAPM alphas			
213	1	1.4***	1.09***	0.76***	-0.21**
214	2	1.57***	1.3***	0.839***	-0.175**
213	3	0.651***	0.609***	0.39***	-0.19*
214	4	0.286	0.274	0.106	-0.271***
213	5	-0.481	-0.0811	-0.315	-0.344***
		Panel C: FF alphas			
213	1	1.52***	1.22***	0.904***	-0.156*
214	2	1.66***	1.4***	0.914***	-0.132***
213	3	0.821***	0.757***	0.461***	-0.155**
214	4	0.441	0.423*	0.286*	-0.221***
213	5	-0.445	-0.00583	-0.156	-0.219***

The table reports the averages of the post-formation period raw and risk-adjusted monthly returns on the 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 months stratified over the quintiles of the CID measure for strong positive and negative price trends (i.e., the $CID_{pos,neg}$ measure). The table also reports the monthly returns of these strategies between months 13 and 60. Quintile five identifies the highest discreteness level. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

To assess the merits of this conjecture, I follow the methodology used in CGH to evaluate the predictive power of the market state on momentum. Presently, I form quintiles of the CID measure and stratify on them the momentum cumulative returns. To com-

Table 6: Stratified Momentum Averages on CID_{neg}-discreteness

N.obs.	CID Qunt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw			
140	1	0.941***	0.808***	0.667***	-0.194*
214	2	0.924***	0.82***	0.521***	-0.164**
213	3	0.806***	0.867***	0.662***	-0.062
214	4	1.22***	0.772**	0.256	-0.146*
213	5	-1.18**	-0.654*	-0.655***	-0.398**
		Panel B: CAPM alphas			
140	1	1.01***	0.851***	0.69***	-0.247**
214	2	1.11***	0.937***	0.58***	-0.221***
213	3	0.906***	0.956***	0.713***	-0.115
214	4	1.28***	0.834***	0.288	-0.206***
213	5	-0.999**	-0.552	-0.603**	-0.476***
		Panel C: FF alphas			
140	1	1.07***	0.965***	0.826***	-0.184**
214	2	1.19***	1.07***	0.718***	-0.156***
213	3	1.08***	1.07***	0.801***	-0.073
214	4	1.43***	0.94***	0.427***	-0.151**
213	5	-0.887**	-0.407	-0.454**	-0.391***

The table reports the averages of the post-formation period raw and risk-adjusted monthly returns on the 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 months stratified on the quintiles of the CID measure for strong negative price trends (i.e., the CID_{neg} measure). The table also reports the monthly returns of these strategies between months 13 and 60. Quintile five identifies the highest discreteness level. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

mence with, I present the results for the $CID_{pos,neg}$ discreteness measure, which gauges the incidence of high discreteness in information shocks generating good and bad news.

As shown in Table 5, the sorting reveals that raw and risk-adjusted momentum gains returns are strongly positive and significant in all but the highest quintiles of the CID measure. Instead, momentum returns are insignificant for the top CID quintile. Echoing the cross-sectional results of DGW, there is an almost monotonic negative relationship between the $CID_{pos,neg}$ measure and momentum gains. Specifically, the profitability of the momentum strategy is broadly inversely related to the incidence of discrete information shocks.

The $CID_{pos,neg}$ measure captures the likelihood with which good and bad news is generated by discrete information shocks during the momentum strategy's formation period. The CID_{neg} measure focuses on the discreteness of bad news. Table 6 shows that a severe concentration of highly discrete bad news ushers economically and statistically significant momentum losses, as shown in Table 6. For instance, over the three-month investment horizon, the momentum strategy loses about 1.18% per month in raw terms. As per Table 5, high discreteness in bad and good news precedes insignificant momentum profits, rather than losses. From this perspective, the CID_{neg} measure is more incisive than $CID_{pos,neg}$ in identifying adverse conditions for momentum investing.[28]

CGH illustrates the market states' predictive power by plotting state-stratified momentum cumulative raw and risk-adjusted returns. I follow their approach in Figure 3 to visualize the effect of high levels of discreteness on momentum profitability. The top panel contrasts the predictive power of high discreteness as captured by $CID_{pos,neg}$ and CID_{neg}, respectively. In the bottom panel, the comparison is between CID_{pos} and CID_{neg}. The figure makes apparent that the effect of high levels of discreteness in negative price trends drives that of high discreteness in positive and negative price trends in determin-

[28]For later use, it is worth mentioning that higher levels of CID_{neg}-discreteness yield an even worse performance, consistent with the FIP hypothesis. For instance, the three-month holding period portfolio yields returns at about -4% per month when CID_{neg} falls above the 97.5th percentile (untabulated results).

ing the combined effect of high discreteness on momentum.

The remaining of this study focuses on the $CID_{pos,neg}$ and CID_{neg} measures. Since $CID_{pos,neg}$ is an average of CID_{pos} and CID_{neg}, contrasting the results for these two measures yields insights into the effect of discreteness in positive price trends.[29]

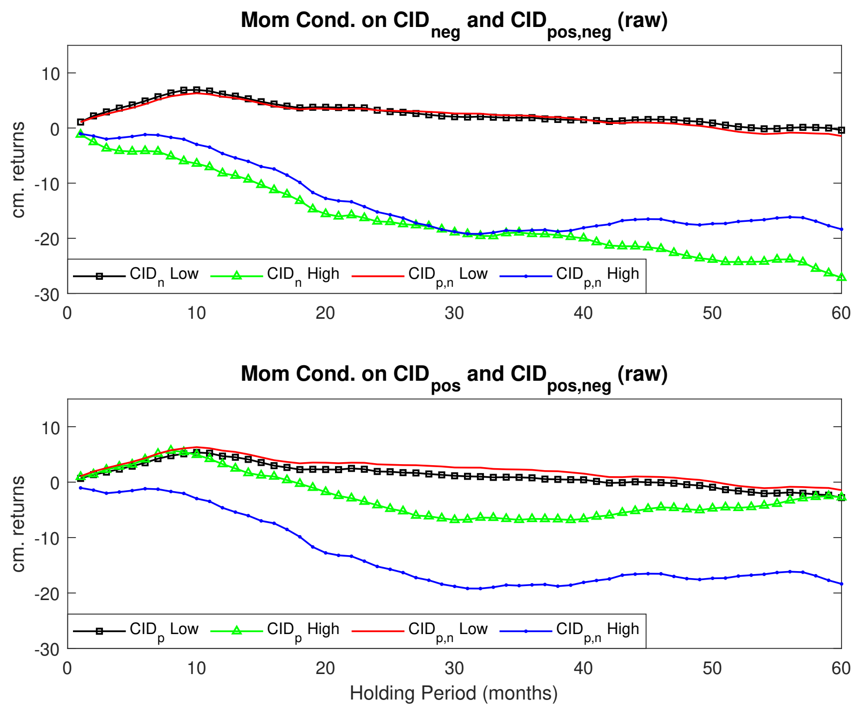

Figure 3: The top panel shows the stratified averages of the cumulative raw returns on the 6m. formation-period momentum strategy for holding periods ranging from one to 60 months, sorted by the $CID_{pos,neg}$ and CID_{neg}-discreteness measures at portfolio formation. High discreteness is captured by the top $CID_{pos,neg}$ and CID_{neg} quintiles, respectively, and low discreteness by the corresponding bottom four quintiles. The bottom panel offers the analogous comparison for the $CID_{pos,neg}$ and CID_{pos}-discreteness measures.

4 The CID- and Market-State Effects on Momentum

The variations in information discreteness across the states documented in Section 2.3 provide a potential explanation of the market-state effect on momentum, in view of the

[29] Section A.4 offers a few comments on the predictive power for momentum of the CID_{pos} measure.

FIP hypothesis. Should discreteness be the main force determining momentum's return dynamics, the market state should become irrelevant once controlling for information discreteness.[30]

Expanding the approach employed to evaluate the market state effect on momentum, I double-sort the time series of momentum returns over the market states and the corresponding quintile groups for CID_{neg} and $CID_{pos,neg}$ measures.[31]

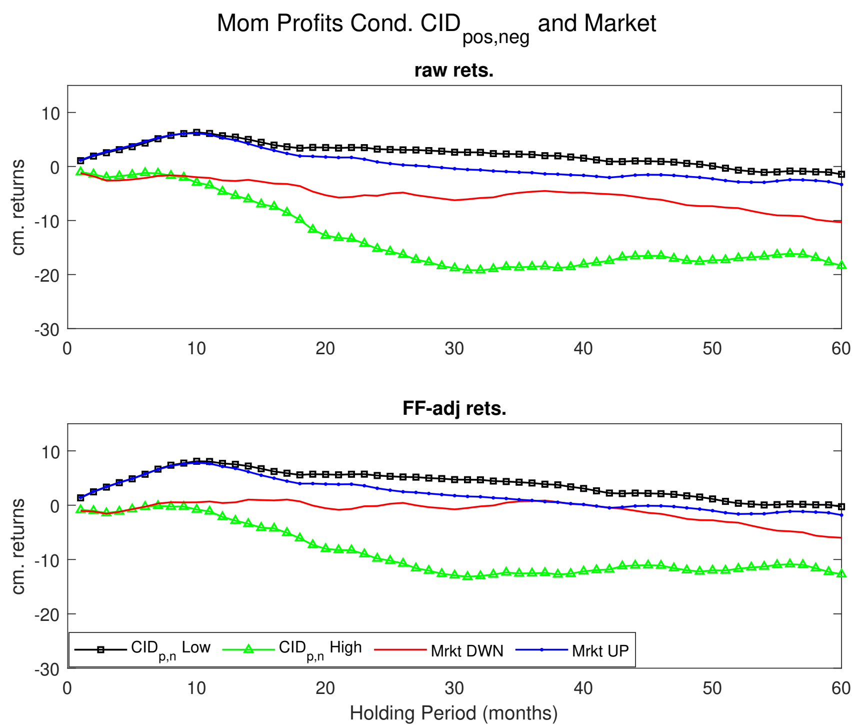

Figure 4: The figure shows the stratified averages of the cumulative raw and Fama-French risk-adjusted returns on the 6m. formation-period momentum strategy for holding periods ranging from one to 60 months, sorted on $CID_{pos,neg}$-discreteness at portfolio formation, and, separately, on the bottom and top four quintiles of the market average return distribution. High discreteness is captured by the top $CID_{pos,neg}$ quintile, and low discreteness by the bottom four $CID_{pos,neg}$ quintiles.

[30] A predictive regression employing the market and the CID measures taken as continuous variables reveals that the predictive power of high discreteness for momentum returns is long-lasting and robust to risk adjustments. However, the results are immaterial to explaining the market-state effect on momentum. Results are in Section A.5 in the Appendix.

[31] Tables 15 and 14 in the Appendix, report the results from this double sorting when the sample of the three-year market returns is partitioned as in CGH, that is, using the zero-return threshold to define the zero-DOWN market state.

Table 7: Market-State Effect (Qnt.le) by the CID_{neg} measure

N.obs.	MRK state	CID Qnt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw				
112	DOWN	1-4	0.296	0.269	0.411**	-0.017
167	2	1-4	1.09***	0.932***	0.726***	-0.126
165	3	1-4	1.7***	1.33***	0.929***	-0.117
166	4	1-4	0.426	0.586***	0.257*	-0.136
171	5	1-4	1.15**	0.798**	0.219	-0.245*
88	1	5	-2.87***	-1.7***	-1.24***	-0.424**
34	2	5	-1.060	-0.321	-0.425	-0.409*
38	3	5	0.501	-0.165	-0.337	-0.292
35	4	5	-0.172	0.529	0.132	-0.364**
18	5	5	1.37**	0.517	-0.410	-0.536***
		Panel B: CAPM alphas				
112	DOWN	1-4	0.437	0.379	0.471**	-0.080
167	2	1-4	1.17***	1.01***	0.768***	-0.171*
165	3	1-4	1.79***	1.41***	0.972***	-0.177**
166	4	1-4	0.624**	0.711***	0.321**	-0.198**
171	5	1-4	1.18**	0.824**	0.231	-0.297**
88	DOWN	5	-2.59***	-1.53***	-1.16***	-0.525***
34	2	5	-0.766	-0.194	-0.373	-0.493**
38	3	5	0.405	-0.171	-0.334	-0.364
35	4	5	0.149	0.668*	0.215	-0.431***
18	5	5	1.16*	0.382	-0.484	-0.522***
		Panel C: FF alphas				
112	DOWN	1-4	0.466	0.466	0.561***	0.012
167	2	1-4	1.24***	0.998***	0.803***	-0.103
165	3	1-4	1.93***	1.5***	1.04***	-0.121
166	4	1-4	0.749***	0.91***	0.496***	-0.179**
171	5	1-4	1.39***	1.03***	0.469**	-0.242**
88	DOWN	5	-2.3***	-1.33***	-0.862***	-0.389**
34	2	5	-1.070	-0.240	-0.506	-0.446**
38	3	5	0.507	-0.011	-0.187	-0.342*
35	4	5	0.108	0.78**	0.405	-0.396***
18	5	5	1.48**	0.648	-0.598	-0.387***

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 m. two-way stratified on the bottom four and top quintiles of the CID measure for strong negative price trends (CID_{neg}) and the market states (quintiles). The table also reports the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

Table 8: Market-State Effect (Qnt.le) by the $CID_{pos,neg}$ measure

N.obs.	MRK state	CID Qnt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw				
136	DOWN	1-4	-0.050	-0.050	-0.010	-0.13
180	2	1-4	0.949***	0.833***	0.634***	-0.112
189	3	1-4	1.54***	1.18***	0.798***	-0.138
183	4	1-4	0.586**	0.74***	0.446***	-0.114
166	5	1-4	0.972***	0.693***	0.359**	-0.253**
77	DOWN	5	-2.33***	-0.894	-0.577	-0.215*
34	2	5	-0.683	-0.042	0.124	-0.337**
24	3	5	1.32*	0.313	0.285	-0.29*
31	4	5	-1.040	-0.333	-0.646*	-0.475***
47	5	5	1.530	0.840	-0.471	-0.25
		Panel B: CAPM alphas				
136	DOWN	1-4	0.089	0.068	0.046	-0.2**
180	2	1-4	1.08***	0.922***	0.681***	-0.169*
189	3	1-4	1.62***	1.26***	0.839***	-0.202**
183	4	1-4	0.798***	0.879***	0.519***	-0.18*
166	5	1-4	1.05***	0.75***	0.39**	-0.313***
77	DOWN	5	-2.03***	-0.711	-0.471	-0.316**
34	2	5	-0.659	-0.031	0.121	-0.376***
24	3	5	1.23*	0.310	0.278	-0.34**
31	4	5	-0.747	-0.243	-0.593*	-0.53***
47	5	5	1.490	0.821	-0.492	-0.247
		Panel C: FF alphas				
136	DOWN	1-4	0.221	0.216	0.213	-0.086
180	2	1-4	1.13***	0.964***	0.721***	-0.11
189	3	1-4	1.77***	1.37***	0.918***	-0.159**
183	4	1-4	0.921***	1.05***	0.678***	-0.167**
166	5	1-4	1.28***	0.951***	0.549***	-0.299***
77	1	5	-1.83**	-0.510	-0.282	-0.217*
34	2	5	-0.948	-0.361	-0.027	-0.241**
24	3	5	1.32*	0.308	0.324	-0.244*
31	4	5	-0.806	-0.008	-0.319	-0.451***
47	5	5	1.530	0.918	-0.179	-0.04

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 m. two-way stratified on the bottom four and top quintiles of the CID measure for strong positive and negative price trends ($CID_{pos,neg}$) and the market states (quintiles). The table also reports the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

Table 7 shows strongly significant momentum gains for all market states when CID_{neg} falls below the 80th percentile. When discreteness falls in the top quintile, the momentum strategy yields strong losses in the DOWN market and generally no momentum gains for the other market states. Risk adjusting reinforces these conclusions. Further, there are generally significant risk-adjusted reversal returns following periods marked by high levels of information discreteness in negative price trends. Hence, the incidence of highly discrete negative news identifies the conditions for which the momentum strategy is unprofitable in the short run and also yields long-term losses, for all market states.

There are two instances of significant momentum gains in the top two market quintiles despite high levels in CID_{neg}. For instance, there are significant momentum returns following the 5th quintile of the lagged market return distribution (i.e.., the top market state) when CID_{neg} falls in its highest quintile (i.e., when negative price trends are associated with severe discreteness).

The top market state is characterized by a large percentage of discrete good news and a low incidence of discrete bad news, as discussed in Section 2.3. Hence, controlling for high discreteness in bad news (i.e., for high levels of CID_{neg}) is unlikely to discriminate momentum gains and losses where these are associated with different discreteness levels in good (rather than bad) news. In fact, as shown in Table 8, when discreteness is gauged by $CID_{pos,neg}$, which captures high information discreteness for both good and bad news, the top market state shows insignificant momentum returns in the high-discreteness category. Other instances of significant momentum gains in the high-discreteness category of CID_{neg} bear similar explanations, as the momentum profits turn insignificant once we control for discreteness in both good and bad news by the $CID_{pos,neg}$.

Taken together, the results of Tables 8 and 7 show that the predictive ability of the DOWN market state for momentum profitability is subsumed by the incidence of high discreteness in the momentum strategy's formation period.

Figure 4 contrasts the effect of high levels of discreteness on momentum profitability and the market states in raw returns (top panel) and Fama-French alphas (bottom panel).[32]

Another feature of the market-state effect is that momentum profits peak at the median state (i.e., state three of the quintile-based states). In Section 2.3, it has already been discussed how the average discreteness of good and bad news in the median state is significantly lower than in the other market states. This finding already provides an explanation for momentum peaking in the median state, in view of the FIP hypothesis. However, disaggregating on discreteness levels, as gauged by the $CID_{pos,neg}$ measure, reveals that these large profits are the result of the aggregation over extremely large momentum profits when discreteness is low and generally insignificant momentum returns for severe-discreteness formation periods, as shown in Table 9. These findings imply that the strong momentum profits associated with the median market state are due to particularly low levels of discreteness in good and bad news rather than to the market state itself.[33]

4.1 Discussion: Overreaction Theories and Investor Attention

Overreaction theories (e.g., Hong and Stein, 1999b; Daniel et al., 1998) share the fundamental prediction that the initial mispricing caused by investors' behavioural biases should be corrected over the long run. CGH notes the consistency of strong momentum profits following prolonged market gains with the predictions of overreaction theories, as buoyant aggregate returns boost the behavioural biases underlying the overreaction effect. From this standpoint, the finding that the momentum strategy yields both short- and long-term losses following protracted market depreciations is indeed puzzling.

The results presented in Tables 8 and 7 corroborate the overreaction explanation of the momentum effect for all the market states as long as the information reaches the market

[32]Figure 6 in the Appendix illustrates the analogous comparison in the CGH framework.

[33]Using the CID_{neg} measure leads to consistent conclusions (untabulated).

Table 9: Momentum in Median Market State by $CID_{pos,neg}$ Quintiles

N.obs.	CID Qnt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw			
58	1	2.11***	1.69***	1.11***	-0.161
53	2	2.11***	1.86***	1.15***	-0.106
44	3	0.802	0.755***	0.579***	-0.247
34	4	0.63	-0.209	0.002	-0.01
24	5	1.32*	0.313	0.285	-0.29*
		Panel B: CAPM alphas			
58	1	2.18***	1.78***	1.16***	-0.228*
53	2	2.33***	2.01***	1.23***	-0.165**
44	3	0.81	0.786***	0.593***	-0.297*
34	4	0.618	-0.212	0.0085	-0.0948
24	5	1.23*	0.31	0.278	-0.34**
		Panel C: FF alphas			
58	1	2.34***	1.89***	1.38***	-0.167
53	2	2.39***	2.1***	1.23***	-0.13*
44	3	1.06**	0.943***	0.536**	-0.244*
34	4	0.737	-0.113	0.142	-0.079
24	5	1.32*	0.308	0.324	-0.244*

The table reports the averages of the median market state's post-formation period unadjusted and risk-adjusted monthly returns on 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 m. stratified on the quintiles of the CID measure for strong positive and negative price trends ($CID_{pos,neg}$). The table also reports the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

with discreteness levels that are sufficiently moderate not heighten investors' attention. In contrast, severe discreteness vanishes short- and long-term momentum profits for all the aggregate conditions summarized by the five market states.

Hence, including discreteness in the discussion of momentum profitability yields the insight that high levels of investors' attention may weaken the effect of the behavioural biases causing investors' overreaction.

5 Momentum Crashes

On average, momentum investing yields economically and statistically significant profits. However, relying on the standard momentum strategy (e.g., Jegadeesh and Titman, 1993) to capitalize on the momentum effect might be challenging due to its occasional severe profitability crashes Daniel and Moskowitz (2016); Barroso and Santa-Clara (2015). The results presented so far show that periods marked by more discrete information shocks herald momentum losses, consistent with the FIP hypothesis. The question is whether equally sharp discreteness peaks predict extreme momentum downturns.

I identify momentum crashes by the lowest 1% of one-month holding-period returns on the 6m formation period strategy. Since there are 1069 one-month momentum returns in the sample, there are eleven of these instances, most of them concentrated in the early years of the sample. Table 10 displays the percentiles of the CID measures in the formation period preceding the momentum crashes.[34] The results show that momentum crashes are invariably preceded by a severe incidence of discrete news, consistent with the FIP hypothesis. Momentum crashes are mostly associated with the discreteness of bad news, which is consistent with the particularly strong effectiveness of the measure CID_{neg} in identifying conditions adverse to momentum investing, as already documented by the

[34] The dates of the most severe momentum losses in Daniel and Moskowitz (2016) generally fall later than those reported in Table 10, as the authors rely on the 12-month formation strategy, which is slower to react to new information. However, the dates coincide when I rely on the 12-month formation-period portfolio, as shown in the online appendix.

comparison of Tables 5 and 6. For instance, the most recent momentum crash of 2009 stems from bad news reaching the market with discreteness around the 98th CID_{neg}-discreteness percentile, a discreteness level not unexpected for those familiar with the events defining the great financial crisis.

Table 10: Momentum Crashes and CID Percentiles

Date	1-m Mom ret	CID_{neg}	CID_{neg}	$CID_{pos,neg}$
1933/4	-51.6	99	4.9	98
1932/8	-43.3	95.7	24.8	93.1
2009/4	-36.6	98.9	9.5	98.1
2001/1	-35.8	88.6	73.6	86.5
1932/7	-34.8	94.8	4.9	87.8
1939/9	-31.3	90.1	16.9	73.3
1931/2	-25.6	96.7	4.9	94.4
2000/5	-25.1	71.7	97.7	95.3
1934/1	-24.9	94.9	72.5	94.1
2000/3	-24.1	78.4	93.4	92.6
1975/1	-21.7	84.5	58	73.7

The table displays the percentiles of the CID measures for the 6-m formation period of strategies yielding one-month momentum returns below the 1st percentile.

6 Extreme Returns and the Market-State Effect

The results presented so far highlight the crucial role of discreteness in predicting momentum profitability. The analysis relies on the CID measure, which is a variation of the discreteness measure proposed in (Da et al., 2014). This section discusses the predictive power of the outlier-based gauge of information discreteness described in Section 1.6.

About 6.6% of the stocks in the cross-section show at least one outlier in the formation period on average over time (see Table 1). However, the incidence of outliers is volatile, as indicated by a standard deviation of about 10%. Table 3 reports that the outlier-based measure and $CID_{pos,neg}$ are strongly correlated, at about 0.7, which suggests that the two discreteness proxies might display a similar degree of predictive power for momentum profitability.

Following the quintile-based method employed to evaluate the effect of the CID measures on momentum, I stratify momentum returns over the outlier-based variable. Table 11 shows that a large share of stocks with at least one outlier in the formation period vanishes momentum profitability.

Table 11: Momentum Stratified Averages on Outlier Discreteness

N.obs.	Out. Qunt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw			
165	1	1.04***	0.867***	0.625***	-0.104
214	2	1.38***	1.25***	0.982***	-0.222***
213	3	0.502**	0.453**	0.175	-0.0908
214	4	0.533*	0.441*	0.214	-0.186
213	5	-0.685	-0.274	-0.416	-0.321***
		Panel B: CAPM alphas			
165	1	1.21***	0.999***	0.693***	-0.184
214	2	1.57***	1.38***	1.05***	-0.274***
213	3	0.551**	0.503***	0.2	-0.145*
214	4	0.676**	0.531**	0.26*	-0.235**
213	5	-0.622	-0.237	-0.396	-0.386***
		Panel C: FF alphas			
165	1	1.41***	1.18***	0.865***	-0.139
214	2	1.65***	1.44***	1.14***	-0.196***
213	3	0.638***	0.598***	0.235*	-0.139***
214	4	0.905***	0.743***	0.457***	-0.201**
213	5	-0.612	-0.174	-0.278	-0.245***

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on the 6m. formation-period momentum strategy with holding periods of 3, 6 and 12 months stratified over the quintiles of the outlier-based discreteness measure. The table also reports the monthly returns between months 13 and 60. The lowest quintile of outlier discreteness indicates formation months with the lowest incidence of stocks with at least one outlier falling in the formation period. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

As shown in Table 12, momentum returns are significant (at the 5% level) only following periods where the incidence of stocks with outliers falls below the 80th percentile, irrespective of the market state. However, the outlier-based variable fails to detect the

Table 12: Market-State Effect (Qnt.le) by Outlier Discreteness

N.obs.	MRK state	Out Qnt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw				
105	DOWN	1-4	0.081	0.175	0.277	-0.201**
173	2	1-4	1.08***	0.906***	0.643***	-0.117
183	3	1-4	1.35***	1.09***	0.801***	-0.133
189	4	1-4	0.575**	0.728***	0.407***	-0.113
156	5	1-4	0.877**	0.572**	0.208	-0.234**
97	DOWN	5	-2**	-0.967	-0.758*	-0.129
32	2	5	-1.42*	-0.528	0.049	-0.418**
20	3	5	2.23***	0.354	-0.026	-0.524**
24	4	5	-1.320	-0.494	-0.644**	-0.568***
40	5	5	2.020	1.430	-0.019	-0.46***
		Panel B: CAPM alphas				
105	DOWN	1-4	0.352	0.387	0.384*	-0.254**
173	2	1-4	1.22***	1.01***	0.695***	-0.164*
183	3	1-4	1.41***	1.15***	0.832***	-0.195**
189	4	1-4	0.783***	0.858***	0.475***	-0.181*
156	5	1-4	0.922**	0.609**	0.225	-0.289***
97	DOWN	5	-1.91***	-0.917	-0.729*	-0.236*
32	2	5	-1.5*	-0.586	0.017	-0.49***
20	3	5	2.05***	0.337	-0.037	-0.573**
24	4	5	-0.983	-0.353	-0.566*	-0.604***
40	5	5	2.090	1.480	0.007	-0.443***
		Panel C: FF alphas				
105	DOWN	1-4	0.478	0.463	0.476**	-0.163
173	2	1-4	1.28***	1.05***	0.733***	-0.101
183	3	1-4	1.59***	1.28***	0.915***	-0.161**
189	4	1-4	0.89***	1.03***	0.65***	-0.169**
156	5	1-4	1.18***	0.842***	0.436*	-0.268***
97	DOWN	5	-1.73***	-0.671	-0.493	-0.113
32	2	5	-1.85**	-1.020	-0.269	-0.39***
20	3	5	1.89***	0.097	-0.199	-0.38*
24	4	5	-0.977	-0.122	-0.382	-0.504***
40	5	5	2.060	1.540	0.261	-0.225***

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on the 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 months two-way stratified on the four combined bottom and top quintiles of the outlier measure and the market quintiles. The highest quintile of the outlier variable identifies formation periods with the highest incidence of stocks with at least one outlier in the formation period. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

momentum profits in DOWN markets uncovered by discreteness in bad news (i.e., the CID_{neg} measure).[35] In this sense, the CID_{neg} measure is more incisive in capturing the effect of high discreteness than the outlier-based variable.[36] Figure 5 visualizes this finding.

While both the outlier-based and CID discreteness measures identify periods ushering significant average momentum gains, the former has a distinct advantage in terms of applicability. The CID measures are calculated from high-frequency data (i.e., daily returns), which poses a formidable obstacle to evaluating information discreteness for markets with infrequent trading, like the corporate bond market. The outlier-based measure of discreteness provides an applicable alternative for markets in which the ID_{DGW} and, thus, the CID measures cannot be reliably evaluated.

7 Conclusions

This study proposes a range of novel aggregate measures of information discreteness based on the discreteness measure of (Da et al., 2014) and the incidence of extreme returns. These measures identify periods of momentum profitability, and discreteness in adverse information shocks is shown to fully subsume the market-state effect on momentum, a longstanding puzzle in the momentum literature. Further findings demonstrate that peaks in information discreteness often forecast dramatic momentum losses. Taken together, the results of this study provide strong corroboration for the FIP hypothesis.

Given this study's results, the causes of the time variations in momentum profitability should be searched among variables linked to information discreteness and, thus, investors' attention. Therefore, the design of momentum strategies that are less prone to

[35]Similar findings apply when the market states are the 0-down and UP, as shown in Table 16 in the Appendix.

[36]The outlier-based discreteness measure is strongly correlated, at 0.71, with $CID_{pos,neg}$, as displayed in Table 3, which is consistent with the finding that these measures display a similar predictive power for momentum returns.

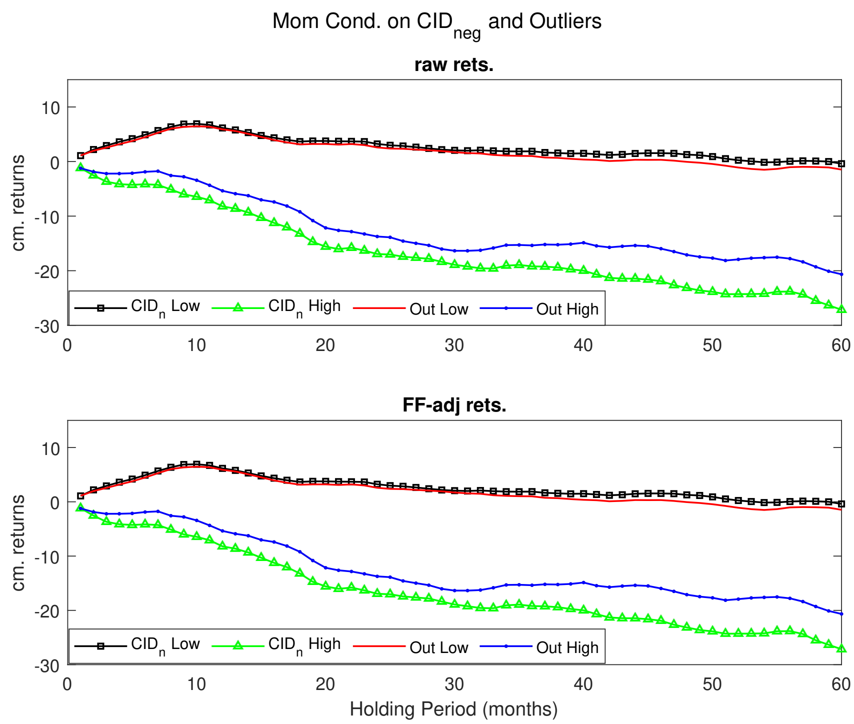

Figure 5: The figure shows the average cumulative momentum returns of the 6m. formation-period strategy with holding periods ranging from one to 60 months, stratified on the bottom four and top fifth quintiles of the outlier-based and CID_{neg} discreteness measures, respectively.

dramatic downturns than the familiar strategy of (Jegadeesh and Titman, 1993) passes through a full understanding of the effect on investors' attention on momentum returns.

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A Appendix

A.1 The Zero-cutoff Market States and Discreteness

Following CGH, at the formation month t the market state is 0-UP (0-DOWN) if the average return of the market portfolio between $t - 36$ and $t - 1$ is greater than zero (less or equal to zero). CGH excludes from the 6-month formation-period momentum portfolios stocks for which the last price of the formation period is below \$1. The results presented in this section (including the CID measures) are calculated after applying the same price filter.[37]

Table 13 documents the market-state effect on momentum, and shows patterns in momentum profits very similar to those uncovered in CGH, including insignificant long-term reversals.

When the DOWN market state is identified by the 20th percentile of the three-year average market return, high discreteness levels are marked by the 80th percentile of the CID measures. The market's zero-return threshold is at about the 13th percentile. For consistency, the predictive power of the 0-DOWN and 0-UP market states are compared with that of the discreteness levels identified by the 87th percentile of the CID measures.

The results in Tables 14 and 15 are consistent with those obtained for quintile-based market states. Specifically, there are momentum gains solely following periods in which bad news does not reach the market with high discreteness, irrespective of the market state.[38] Severe discreteness in bad news makes momentum unprofitable in both 0-UP and 0-DOWN markets. Hence, the measure CID_{neg} fully subsumes the 0-DOWN and 0-UP market state effect on momentum. Discreteness in good and bad news and the incidence of outliers are less incisive in discriminating momentum gains and losses in 0-DOWN markets. Figure 6 contrasts the predictive power of discreteness in bad news and the CGH market state effect.

A.2 The DGW ID Measure and Momentum

Following DGW, I double-sort the equity cross-section on the quintiles of the formation-period cumulative returns (i.e., the information content) and then on quintiles of the ID_{DGW} discreteness measure, for each month. Note that stocks are ranked on ID_{DGW}

[37]Specifically, formation periods showing a last price lower than \$1 do not contribute to the calculations of the CID measures.

[38]Momentum profits in the 0-DOWN state with low discreteness levels are strongly significant in risk-adjusted terms.

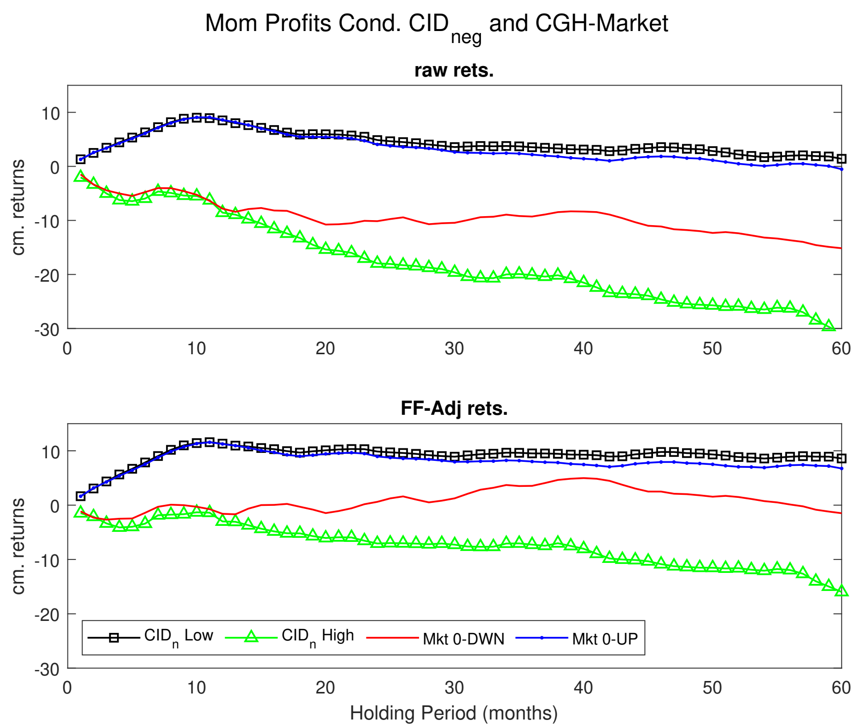

Figure 6: The figure shows the stratified averages of the cumulative raw and Fama-French risk-adjusted returns on the 6m. formation-period momentum strategy for holding periods ranging from one to 60 months, sorted on the quantiles defined by the 87th percentile of the CID_{neg}-discreteness at portfolio formation and on the market states as in Cooper et al. (2004). Price filter as in Cooper et al. (2004).

Table 13: Market State Effect (Zero Cut-off)

N.obs.	MRK Qunt.le	3m.	6m.	12m.	13-60m
		Panel A: Raw			
143	0-DOWN	-1.62**	-0.872	-0.7**	-0.150
926	0-UP	1.13***	1.02***	0.716***	-0.19**
		Panel B: CAPM alphas			
143	0-DOWN	-1.25**	-0.555	-0.447	-0.133
926	0-UP	1.32***	1.15***	0.818***	-0.185**
		Panel C: FF alphas			
143	0-DOWN	-0.99*	-0.301	-0.160	-0.001
926	0-UP	1.42***	1.26***	0.937***	-0.094

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on the 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 months stratified over the market states defined as in Cooper et al. (2004). The table also reports the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

levels within each cumulative return quintile, as a comparison of information discreteness is meaningful only if controlling for the level of information content, which is empirically gauged by the formation-period return. Next, standard 6-m. formation-period momentum portfolios (e.g., Jegadeesh and Titman, 1993), with a skip month, are formed by matching the ID_{DGW} levels of winners and losers.

The negative relationship between ID and momentum uncovered in Da et al. (2014) holds in this study's sample, as shown in Table 17.

Table 14: CGH Market-State Effect by CID_{neg}-discreteness

N.obs.	MRK state	CID Qnt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw cum. rets.				
89	0-DOWN	low CID	-0.122	0.521	0.351*	-0.006
778	0-UP	low CID	1.3***	1.11***	0.75***	-0.165**
52	0-DOWN	high CID	-4.19**	-3.37***	-2.48***	-0.449**
87	0-UP	high CID	-0.161	0.426	0.344	-0.478**
		Panel B: CAPM-alpha				
89	0-DOWN	low CID	0.219	0.823**	0.567***	0.015
778	0-UP	low CID	1.46***	1.23***	0.845***	-0.16*
52	0-DOWN	high CID	-3.74***	-3.01***	-2.15***	-0.44**
87	0-UP	high CID	0.094	0.567	0.451	-0.465**
		Panel C: FF-alpha				
89	0-DOWN	low CID	0.333	0.958**	0.643***	0.112
778	0-UP	low CID	1.58***	1.35***	0.978***	-0.076
52	0-DOWN	high CID	-3.16***	-2.55***	-1.45***	-0.241
87	0-UP	high CID	0.100	0.617	0.465	-0.288*

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 m. two-way stratified by the quantiles defined by the 87the percentile of the CID measure for strong negative price trends (CID_{neg}) and the market states defined in Cooper et al. (2004). The table also reports the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

Table 15: CGH Market-State Effect by $CID_{pos,neg}$-discreteness

N.obs.	MRK state	CID Qnt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw cum. rets.				
85	0-DOWN	low-CID	0.064	-0.456	-0.339	-0.188*
844	UP	low-CID	1.36***	1.11***	0.767***	-0.151*
58	0-DOWN	high-CID	-4.08***	-1.480	-1.23**	-0.094
81	UP	high-CID	-1.29*	0.100	0.195	-0.603***
		Panel B: CAPM-alpha				
85	0-DOWN	low-CID	0.162	-0.279	-0.180	-0.167
844	UP	low-CID	1.55***	1.25***	0.875***	-0.144*
58	0-DOWN	high-CID	-3.31***	-0.960	-0.838	-0.083
81	UP	high-CID	-1.12*	0.171	0.243	-0.616***
		Panel C: FF-alpha				
85	0-DOWN	low-CID	0.329	-0.091	0.042	-0.025
844	UP	low-CID	1.67***	1.37***	1***	-0.074
58	0-DOWN	high-CID	-2.92**	-0.609	-0.455	0.034
81	UP	high-CID	-1.15*	0.065	0.252	-0.297***

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 m. two-way stratified on the quantiles defined by the 87the percentile of the CID measure for strong positive and negative price trends ($CID_{pos,neg}$) and the market states defined in Cooper et al. (2004). The table also reports the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

Table 16: CGH Market-State Effect by Outlier Discreteness

N.obs.	MRK state	Out. Qnt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw cum. rets.				
74	0-DOWN	low	-1	-0.606	-0.340	-0.243*
821	UP	low	1.18***	1.02***	0.721***	-0.17**
68	0-DOWN	high	-2.31*	-1.170	-1.09**	-0.050
71	UP	high	0.450	0.993	0.696	-0.446***
		Panel B: CAPM-alpha				
74	0-DOWN	low	-0.452	-0.169	-0.036	-0.231*
821	UP	low	1.37***	1.15***	0.831***	-0.163*
68	0-DOWN	high	-2.15*	-0.992	-0.903*	-0.027
71	UP	high	0.514	0.977	0.642	-0.464***
		Panel C: FF-alpha				
74	0-DOWN	low	-0.228	-0.054	0.162	-0.095
821	UP	low	1.47***	1.26***	0.946***	-0.092
68	0-DOWN	high	-1.84*	-0.577	-0.507	0.101
71	UP	high	0.649	1.1*	0.807*	-0.125

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 months two-way stratified on low and high-discreteness quantiles defined by the 87th percentile of the outlier measure and the market states defined in Cooper et al. (2004). The highest quantile of the outlier variable indicates formation months preceded by the highest incidence of stocks with outliers in the formation period. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

Table 17: Unconditional Momentum is Decreasing in ID_{DGW} Discreteness

N.obs.	ID Qunt.le	3m.	6m.	12m.	13-60m
		Panel A: Raw			
1057	5	-0.017	0.145	0.135	-0.0825**
1051	4	0.044	0.191	0.138	-0.0906*
1052	3	0.244*	0.305***	0.212**	-0.115**
1059	2	0.397***	0.463***	0.296***	-0.136**
1068	1	0.636***	0.581***	0.366***	-0.171**
		Panel B: CAPM alphas			
1057	5	0.009	0.173	0.152	-0.0918**
1051	4	0.103	0.218*	0.136	-0.115**
1052	3	0.319**	0.374***	0.254***	-0.166***
1059	2	0.47***	0.525***	0.335***	-0.206***
1068	1	0.795***	0.717***	0.45***	-0.247***
		Panel C: FF alphas			
1057	5	0.065	0.242**	0.225**	-0.0522
1051	4	0.198	0.314***	0.233**	-0.0629
1052	3	0.425***	0.484***	0.365***	-0.116***
1059	2	0.565***	0.633***	0.457***	-0.154***
1068	1	0.913***	0.843***	0.578***	-0.188***

The table reports the average post-formation period monthly returns on the 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 months. The table also reports the monthly returns between months 13 and 60. The strategies are defined by a sequential double sorting involving the formation-period return and the ID_{DGW} discreteness measure as in Da et al. (2014). The 5th quintile identifies the highest incidence of high discreteness information. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

Table 18: ID_{DGW} and Momentum in DOWN Markets

N.obs.	ID_{DGW} Qunt.le	3m.	6m.	12m.	13-60m
		Panel A: Raw			
213	1	-0.473	-0.241	-0.305	-0.252***
207	2	-0.722**	-0.279	-0.223	-0.218*
203	3	-0.83**	-0.439	-0.211	-0.138**
203	4	-1.02**	-0.401	-0.090	-0.073
206	5	-0.695*	-0.303	-0.186	-0.051
		Panel B: CAPM alphas			
213	1	-0.240	-0.027	-0.173	-0.355***
207	2	-0.63*	-0.193	-0.166	-0.308**
203	3	-0.716*	-0.330	-0.146	-0.205***
203	4	-0.955**	-0.374	-0.099	-0.105
206	5	-0.664*	-0.264	-0.161	-0.060
		Panel C: FF alphas			
213	1	-0.017	0.228	0.066	-0.224***
207	2	-0.505	-0.034	0.010	-0.199*
203	3	-0.502	-0.153	0.018	-0.127**
203	4	-0.959**	-0.333	-0.021	-0.033
206	5	-0.698*	-0.277	-0.147	-0.020

The table reports the post-formation period average monthly returns of the 6m. formation-period momentum strategies within discreteness groups for holding periods of 3, 6 and 12 months when the formation month is in the DOWN market state. The strategies are defined by a sequential double sorting involving the formation-period return and ID_{DGW}-discreteness as in (Da et al., 2014). The table also reports the monthly returns between months 13 and 60. The 5th quintile identifies the highest ID_{DGW}-discreteness level. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

A.3 Regression Results: Incidence of High Discreteness in DOWN, Median, top Markets

I regress the variables $HighID_{b,t}$ for $b \in \{1, 2, 3, 18, 19, 20\}$ over a constant and four market-state dichotomous variables assuming values 0 and 1 for the market quintiles 2, 3, 4, and 5, respectively, where the first market quintile is the DOWN state. The regression is also evaluated for the conditional information discreteness measures $CID_{pos,neg}$ and

Table 19: ID_{DGW} and Momentum in UP Markets

N.obs.	ID_{DGW} Qunt.le	3m.	6m.	12m.	13-60m
		Panel A: Raw			
855	1	0.912***	0.786***	0.533***	-0.15*
852	2	0.669***	0.644***	0.422***	-0.117**
849	3	0.5***	0.483***	0.313***	-0.109**
848	4	0.299**	0.333***	0.193*	-0.095**
851	5	0.147	0.254**	0.213*	-0.090**
		Panel B: CAPM alphas			
855	1	1.05***	0.903***	0.605***	-0.22***
852	2	0.738***	0.699***	0.457***	-0.181***
849	3	0.567***	0.543***	0.349***	-0.157***
848	4	0.357**	0.359***	0.192*	-0.117***
851	5	0.172	0.278**	0.228**	-0.099**
		Panel C: FF alphas			
855	1	1.14***	0.996***	0.706***	-0.179***
852	2	0.825***	0.795***	0.565***	-0.143***
849	3	0.646***	0.637***	0.447***	-0.114***
848	4	0.475***	0.469***	0.295***	-0.070*
851	5	0.25*	0.367***	0.315***	-0.060

The table reports the post-formation period average monthly returns of the 6m. formation-period momentum strategies within discreteness groups for holding periods of 3, 6 and 12 months when the formation month is in the UP market state (i.e., the combined top four quintiles of lagged market three-year returns). The strategies are defined by a sequential double sorting involving the formation-period return and ID_{DGW}-discreteness as in (Da et al., 2014). The table also reports the monthly returns between months 13 and 60. The 5th quintile identifies the highest ID_{DGW}-discreteness level. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

CID_{neg}.[39] The same methodology yields an assessment of the relative incidence of high discreteness in the central and top market states. The regression results are in Tables 20, 21, and 22, for the DOWN, median, and central market states, respectively.

Table 20: Excess High-discreteness Relative to DOWN markets

Panel A: Excess Incidence High ID by return band						
Market State	1	2	3	18	19	20
2	-1.64***	-1.52***	-1.15***	0.0176	-0.216**	-1.44*
3	-1.86***	-1.62***	-1.1***	-0.147**	-0.5***	-1.6**
4	-1.4***	-1.26***	-0.982***	-0.0504	-0.318	-1.47
5	-2.11***	-1.98***	-1.54***	0.391***	0.283	-0.546
Panel B: Excess CID relative to DOWN market						
Market State	$CID_{pos,neg}$	CID_{neg}				
2	-0.992***	-1.44***				
3	-1.14***	-1.53***				
4	-0.914***	-1.21***				
5	-0.915**	-1.87***				

Panel A reports the coefficients from a regression of $HighID_{b,t}$ over a constant and four indicator variables for the market quintiles 2, 3, 4, and 5, respectively, where the first market quintile is the DOWN market state. The constant's coefficient is left unreported. The regression is evaluated for each return band with $b \in 1, 2, 3, 18, 19, 20$. Panel B reports the coefficient of the analogous regression for the $CID_{pos,neg}$ and CID_{neg} measures.Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

[39]Standard errors are corrected for five lags to account for overlapping formation periods (e.g., Gallant, 1987).

Table 21: Excess High-discreteness Relative to the Central Market State

Panel A: Excess Incidence High ID by return band						
Market State	1	2	3	18	19	20
1	1.75***	1.6***	1.11***	0.162**	0.501***	1.62**
2	0.162**	0.0838	-0.0499	0.172*	0.283***	0.164***
4	0.402***	0.345***	0.122	0.103	0.182	0.135
5	-0.308*	-0.37***	-0.433***	0.544***	0.783***	1.06***
Panel B: Excess CID relative to Central Market State						
Market State	$CID_{pos,neg}$	CID_{neg}				
1	1.12***	1.48***				
2	0.136***	0.0654				
4	0.215***	0.29***				
5	0.213	-0.37***				

Panel A reports the coefficients from a regression of $HighID_{b,t}$ over a constant and four indicator variables for the market quintiles 1, 2, 4, and 5, respectively, where the first market quintile is the DOWN market state. The constant's coefficient is left unreported. The regression is evaluated for each return band with $b \in 1, 2, 3, 18, 19, 20$. Panel B reports the coefficient of the analogous regression for the $CID_{pos,neg}$ and CID_{neg} measures.Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

A.4 CID for Positive Price Trends and Momentum

Table 23 demonstrates the discriminatory power of the CID_{pos}-discreteness measure for momentum profits. By the high correlation levels (reported in Table 3), both CID_{pos} and CID_{neg} should show a predictive power for momentum returns similar to $CID_{pos,neg}$, consistent with this latter being an average of the former measures. While sorting momentum returns on CID_{neg} yields results consistent with the FIP hypothesis, with high discreteness being associated with no momentum gains, the measure CID_{pos} yields mixed results, as it fails to differentiate momentum losses and gains. The implication is that ignoring discreteness in bad news weakens the predictive power of discreteness for momentum returns.

Table 22: Excess High-discreteness Relative to the Top Market Quintile

Panel A: Excess Incidence High ID by return band						
Market State	1	2	3	18	19	20
1	2***	1.95***	1.54***	-0.373***	-0.279	0.57
2	0.41**	0.436***	0.38***	-0.363	-0.497*	-0.886**
3	0.189	0.335***	0.429***	-0.527***	-0.781***	-1.04***
4	0.65***	0.697***	0.552***	-0.431***	-0.598***	-0.916***

Panel B: Excess CID relative to Top markets		
Market State	$CID_{pos,neg}$	CID_{neg}
1	0.9**	1.83***
2	-0.0866	0.409***
3	-0.233*	0.318***
4	-0.0077	0.633***

Panel A reports the coefficients from a regression of $HighID_{b,t}$ over a constant and four indicator variables for the market quintiles 1, 2, 3, and 4, respectively, where the first market quintile is the DOWN market state. The constant's coefficient is left unreported. The regression is evaluated for each return band with $b \in 1, 2, 3, 18, 19, 20$. Panel B reports the coefficient of the analogous regression for the $CID_{pos,neg}$ and CID_{neg} measures.Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

A.5 CID and Market as Continuous Variables

The results presented insofar analyze the predictive power of quantiles of the discreteness measures for momentum returns. An alternative approach is to evaluate a predictive regression relying on the market and discreteness measures as continuous variables. Presently, momentum returns are regressed over a CID variable and the three-year market returns. The regression includes a constant and the squared market variable, as CGH documents its significance in an analogous exploration. The model is:

$$MOM_{t,h} = \alpha + \beta_1 * MRK_t + \beta_2 * MRK_t^2 + \beta_3 * CID + \epsilon_t \tag{1}$$

where $MOM_{t,h}$ is the cumulative return of the momentum strategies formed at time t for holding period of three, six, and twelve months, the variable MRK_t is the three-year

Table 23: Stratified Momentum Averages on CID_{pos}-discreteness

N.obs.	CID Qunt.le	3m.	6m.	12m.	13-60m.
		Panel A: Raw			
161	1	-0.067	-0.014	-0.032	-0.198
213	2	1.03***	0.871***	0.442***	-0.184**
213	3	0.667***	0.669***	0.527***	-0.063
214	4	0.676***	0.7***	0.558***	-0.191***
214	5	0.749*	0.685**	0.272	-0.125
		Panel B: CAPM alphas			
161	1	0.11	0.107	0.0292	-0.275**
213	2	1.2***	0.987***	0.499***	-0.251***
213	3	0.832***	0.786***	0.591***	-0.123*
214	4	0.709***	0.74***	0.581***	-0.252***
214	5	0.819*	0.731**	0.294	-0.162*
		Panel C: FF alphas			
161	1	0.181	0.242	0.144	-0.212**
213	2	1.17***	1.02***	0.596***	-0.217***
213	3	1.01***	0.87***	0.631***	-0.0965**
214	4	0.915***	0.932***	0.706***	-0.197***
214	5	0.914**	0.87***	0.496**	-0.0515

The table reports the averages of the post-formation period unadjusted and risk-adjusted monthly returns on 6m. formation-period momentum strategy for holding periods of 3, 6 and 12 months stratified on the bottom four and top quintiles of the CID measure for strong positive price trends (CID_{pos}). The table also reports the returns between months 13 and 60. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.

average of the monthly returns from $t-36$ to $t-1$, CID is the conditional discreteness measure for price trends over the momentum strategy's formation period from $t-5$ and $t-1$, and ϵ_t is a zero-mean error term. The results are reported for holding period returns of three, six, and twelve months. Standard errors are corrected for heteroskedasticity and autocorrelation (HAC) with lags equal to the number of overlapping months in the holding period window.

Table 24 reports the coefficients of interest for Model 1 together with the R^2 of the regression.[40] As we can see from Panel B, generally both the market index and the CID_{neg} measure (Panel B) are significant for the three-month holding periods raw momentum returns. However, for strategies with longer holding periods, risk adjustment brings about a stronger predictive power for the discreteness variable than the market. Relying on $CID_{pos,neg}$ to gauge discreteness in good and bad news reveals similar predictive patterns (Panel A), albeit there is a weaker significance of discreteness for the shortest investment horizon. Overall, the discreteness variables are more effective than the market in predicting momentum's CAPM and Fama-French alphas for holding periods beyond the very short term.

[40]The levels of model fit reported in Table 24 are similar to the one reported in CGH, with R^2 at about 10%.

Table 24: Predictive Regression Market and CID as Continuous Variables

	Raw			CAPM-adj			FF-adj		
HP	3	6	12	3	6	12	3	6	12
				Panel A CID_pos,neg					
MKT	1.05***	0.703**	0.406**	0.935***	0.622**	0.365*	0.735***	0.493*	0.239
MKT$\hat{2}$	-0.287*	-0.209*	-0.179**	-0.229*	-0.173	-0.16*	-0.097	-0.076	-0.067
$CID_{pos,neg}$	-0.447***	-0.224	-0.200	-0.467***	-0.239	-0.207	-0.542***	-0.279*	-0.236*
R_sq	0.1	0.08	0.09	0.09	0.07	0.08	0.08	0.06	0.06
				Panel B CID_neg					
MKT	0.982***	0.615**	0.345**	0.887***	0.545*	0.309*	0.719***	0.426*	0.192
MKT$\hat{2}$	-0.325**	-0.223**	-0.193***	-0.27**	-0.190	-0.176**	-0.148	-0.098	-0.086
CID_{neg}	-0.393***	-0.265***	-0.213***	-0.375***	-0.26***	-0.212***	-0.386***	-0.273***	-0.219***
R_sq	0.13	0.12	0.15	0.11	0.11	0.13	0.1	0.08	0.08

The table reports the coefficients and the R^2 for Model 1 for the raw and risk-adjusted monthly returns of momentum strategies with holding periods of 3, 6 and 12 months. Panels A and B list the market and CID coefficients when discreteness is gauged by $CID_{pos,neg}$ and CID_{neg}, respectively. The intercept coefficients are omitted. Significance at the 0.01, 0.05 and 0.1 levels are marked by ***, **, and *, respectively.